Tutorial #2

MAT 188 – Linear Algebra I – Fall 2015

SOLUTIONS

Problems (Please note these are from Holt's Linear Algebra Text)

1.2 #21: Calculation Convert the system to an augmented matrix and then find all solutions by reducing the system to echelon form and back substituting.

$$\begin{array}{rcrcrcr} -2x_1 & + & 5x_2 & - & 10x_3 & = & 4 \\ x_1 & - & 2x_2 & + & 3x_3 & = & -1 \\ 7x_1 & - & 17x_2 & + & 34x_3 & = & -16 \end{array}$$

Solution First we write the augmented matrix.

$$\begin{array}{rcrcrcr} -2x_1 & + & 5x_2 & - & 10x_3 & = & 4 \\ x_1 & - & 2x_2 & + & 3x_3 & = & -1 \\ 7x_1 & - & 17x_2 & + & 34x_3 & = & -16 \end{array} \implies \left(\begin{array}{ccc|c} -2 & 5 & -10 & 4 \\ 1 & -2 & 3 & -1 \\ 7 & -17 & 34 & -16 \end{array}\right)$$

Now row reduce the matrix. You'll find

$$\begin{aligned} \left(\begin{array}{ccc|c} -2 & 5 & -10 & 4 \\ 1 & -2 & 3 & -1 \\ 7 & -17 & 34 & -16 \end{array}\right) &\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ -2 & 5 & -10 & 4 \\ 7 & -17 & 34 & -16 \end{array}\right) \\ &\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 0 & -3 & 13 & -9 \end{array}\right) \\ &\sim \left(\begin{array}{ccc|c} 1 & 0 & -5 & 3 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right) \\ &\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -12 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & -3 \end{array}\right) \end{aligned}$$

Thus we read the solution off the reduced augmented matrix as

$$\boxed{x_1 = -12 \quad x_2 = -10 \quad x_3 = -3}$$

□

1.3 #25: Calculation Compute the first four Gauss-Seifel iterations for the system, with the initial value of each variable set equal to 0. Then rewrite the system so that it is diagonally dominant, set the value of each variable to 0, and again compute four Gauss-Seidel iterations.

$$\begin{array}{rcrcr} x_1 & - & 2x_2 & = & -1 \\ 2x_1 & - & x_2 & = & 1 \end{array}$$

Solution We set $x_1^0 = 0, x_2^0 = 0$ and then use the Gauss-Seifel iterations, i.e.

$$x_1^{i+1} = -1 + 2x_2^i$$

$$x_2^{i+1} = -1 + 2x_1^{i+1}$$

Iteration	x_1	x_2
0	0	0
1	-1	-3
2	-7	-15
3	-31	-63
4	-127	-255

i.e. this doesn't converge...But if we reorder the equations to

$$\begin{array}{rcrcr} 2x_1 & - & x_2 & = & 1 \\ x_1 & - & 2x_2 & = & -1 \end{array}$$

then it is diagonally dominant, so we see

$$x_1^{i+1} = \frac{1 + x_2^i}{2}, \quad x_2^{i+1} = \frac{1 + x_1^{i+1}}{2}$$

Iteration	x_1	x_2
0	0	0
1	1/2	3/4
2	7/8	15/16
3	31/32	63/64
4	127/128	255/256

i.e. the solution is converging to $(1, 1)$. $\square$

1.4 #25 Calculation The equation for a parabola has the form $y = ax^2 + bx + c$, where a, b, and c are constants and $a \neq 0$. Find an equation for the parabola that passes through the points $(-1, -10), (1, -4)$, and $(2, -7)$.

Solution By inserting the required points to pass through, we obtain 3 equations with 3 unknowns, namely.

$$\begin{array}{ccccccc} a & - & b & + & c & = & -10 \\ a & + & b & + & c & = & -4 \\ 4a & + & 2b & + & c & = & -7 \end{array} \implies \left(\begin{array}{ccc|c} 1 & -1 & 1 & -10 \\ 1 & 1 & 1 & -4 \\ 4 & 2 & 1 & -7 \end{array}\right)$$

Now we reduce the system. We see

$$\begin{aligned} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -10 \\ 1 & 1 & 1 & -4 \\ 4 & 2 & 1 & -7 \end{array}\right) &\sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & -10 \\ 0 & 2 & 0 & 6 \\ 0 & 6 & -3 & 33 \end{array}\right) \\ &\sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & -10 \\ 0 & 1 & 0 & 3 \\ 0 & -2 & 1 & -11 \end{array}\right) \\ &\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right) \end{aligned}$$

Thus the parabola $y = -2x^2 + 3x - 5$ intersects the 3 required points. □

2.1 #29: Calculation Determine if b is a linear combination of the other vectors. If so, write b as a linear combination.

$$a_1 = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 0 \\ 3 \\ -3 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 3 \\ -9 \end{pmatrix}$$

Solution We have to see if

$$x_1 a_1 + x_2 a_2 = b, \quad x_1, x_2 \in \mathbb{R}$$

has a solution. Reading off the first component we see

$$2x_1 = 6 \implies \boxed{x_1 = 3}$$

Reading off the second component we see

$$-3x_1 + 3x_2 = 3 \implies -9 + 3x_2 = 3 \implies \boxed{x_2 = 4}$$

Now we check to make sure the third component is consistent with our choice of x_1 and x_2.

$$x_1 - 3x_2 = -9 \implies 3 - 12 = -9 \implies -9 = -9$$

Since the values check out, we see

$$\boxed{3a_1 + 4a_2 = b}$$

□