# Tutorial #3

MAT 188 – Linear Algebra I – Fall 2015

SOLUTIONS

**Problems** (Please note these are from Holt's Linear Algebra Text)

**1.4 #21: Calculation** Find the values of the missing constants.

$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

**Solution** To create a system of equations for $A, B$ and $C$, rearrange the RHS to a common denominator.

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} = \frac{Ax(x-1) + B(x-1) + Cx^2}{x^2(x-1)}$$

Now if we read of the numerator of the LHS and the rearranged RHS we see

$$Ax(x-1) + B(x-1) + Cx^2 = 1 \implies \begin{cases} A + C = 0 \\ -A + B = 0 \\ -B = 1 \end{cases}$$

The solution to this system of equations is easily seen to be

$$\boxed{A = B = -1, \quad C = 1}$$

□

**2.1 #49: Calculation** An electronics company has two production facilities, A and B. During an average week, facility A produces 2000 computer monitors and 8000 flat panel televisions, and facility B produces 3000 computer monitors and 10,000 flat panel televisions.

1. Give vectors $a$ and $b$ that give the weekly production amounts at A and B, respectively.

$$\boxed{\text{Answer:} \qquad a = \begin{pmatrix} 2000 \\ 8000 \end{pmatrix} \quad \& \quad b = \begin{pmatrix} 3000 \\ 10,000 \end{pmatrix}}$$

2. Compute $8b$, and then describe what the entries tell us.

$$\boxed{\text{Answer:} \qquad 8b = 8\begin{pmatrix} 3000 \\ 10,000 \end{pmatrix} = \begin{pmatrix} 24,000 \\ 80,000 \end{pmatrix}}$$

The entires tell us that 24,000 computer monitors and 80,000 flat panel televisions will be produced by facility B over 8 weeks.

3. Determine the combined output from A and B over a 6-week period.

$$\text{Answer:} \qquad 6(a+b) = 6\begin{pmatrix} 5000 \\ 18,000 \end{pmatrix} = \begin{pmatrix} 30,000 \\ 108,000 \end{pmatrix}$$

4. Determine the number of weeks of production from A and B required to produce 24,000 monitors and 92,000 televisions.

**Solution** We see this produces the following linear system.

$$x_1 a + x_2 b = \begin{pmatrix} 24,000 \\ 92,000 \end{pmatrix}$$

The solution to which may be easily found to be

$$\boxed{x_1 = 9, \quad x_2 = 2}$$

Thus is we run facility A for 9 weeks and facility B for 2 weeks we'll produce the product required. □

**2.2 # 71** Prove that if $\text{span}\{u_1, u_2, u_3\} = \mathbb{R}^3$, then

$$\text{span}\{u_1 + u_2, u_1 + u_3, u_2 + u_3\} = \mathbb{R}^3$$

**Proof** Without the loss of generality we may assume

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

since the original vectors form a basis for $\mathbb{R}^3$. If suffices to check that we have

$$\text{span}\{u_1, u_2, u_3\} = \text{span}\{u_1 + u_2, u_1 + u_3, u_2 + u_3\}$$

which is equivalent to checking if the RHS has full rank, i.e.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

This is true since

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

□