MAT244 - Ordinary Differential Equations - Summer 2014
Assignment 1 Due: May 28, 2014

Full Name: ______________________________________

Last First

Student #: ______________________________________

Indicate which Tutorial Section you attend by filling in the appropriate circle:

◯	Tut 01	W 12:10-13:00	WI 523	Christopher Adkins
◯	Tut 02	W 17:10 - 16:00	LM 158	Yuri Cher
◯	Tut 03	W 17:10 - 16:00	SS 1074	Alexander Caviedes

Instructions:

- Due May 28, 2014 before the lecture at 13:10pm in MP203.
- Assignments should be completed individually. If you want to work in groups, work on similar textbook problems but do the assignment problems on your own.
- Write your solutions clearly, showing all steps. The solutions presented should not be your first draft! Grading is based on correctness as well as presentation.
- Assignments may be submitted up to one week after their due date, scanned, or carefully photographed, and submitted in a PDF via email to the course instructor: craig.sinnamon@utoronto.ca
- There will be a 5% reduction in mark per day of lateness beginning after 13:10pm on the day the assignment is due.
- Assignments may be submitted to the course instructor for remarking during office hours. Assignments that do not meet the following criterion may not be accepted for remarking.
  - the assignment was returned less than **EIGHT** days ago
  - the assignment is written in pen
  - the assignment is accompanied with an attached note clearly explaining the grading complaint
- Note that grades may decrease after remarking.

1. Solve the initial value problem

$$\begin{cases} y'(t) - 2y(t) = 2 \\ y(0) = 1 \end{cases}$$

$$y' = 2 + 2y \iff \int \frac{dy}{1+y} = \int 2\,dt$$

$$\iff \ln(1+y) = 2t + C$$

$$\iff y(t) = \tilde{C}e^{2t} - 1$$

$$y(0) = 1 \implies \tilde{C} = 2$$

$$\therefore \boxed{y(t) = 2e^{2t} - 1}$$

2. a) State the order of the equation $y''(t) - y'(t) = \tan(t)$, and whether it is linear or nonlinear.

Order $= 2$, linear

b) Verify that $y_1(t) = \ln(\sec(t) + \tan(t))$ is a solution to $y''(t) - \tan(t)y'(t) = 0$.

$y' = \sec(t)$

$y'' = \tan(t)\sec(t)$

$\Rightarrow$ L.H.S $=$ R.H.S ✓

JW

3. Find the general solution of the differential equation below, and use it to determine the behaviour of solutions as $t \to \infty$.

$$ty'(t) - y(t) = t - 3, \quad t > 0$$

$\iff y' - \frac{1}{t}y = 1 - \frac{3}{t}$ notice $I(t) = \exp\left(-\int \frac{1}{t}\,dt\right) = \frac{1}{t}$

$\iff \left(\frac{1}{t}y\right)' = \frac{1}{t} - \frac{3}{t^2}$

$\iff y(t) = t \int \frac{1}{t} - \frac{3}{t^2}\,dt = t\left(\ln(t) + \frac{3}{t} + C\right)$

$= Ct + t\ln(t) + 3$

$= t(C + \ln(t)) + 3$

$\therefore$ if $t \to \infty$, then $y \to \infty$ $\forall C \in \mathbb{R}$

4. The population of mosquitoes in a certain area increases at a rate proportional to the current population, and in the absence of other factors, the population doubles each week. There are 200,000 mosquitoes in the area initially, and predators (birds, bats, and so forth) eat 20,000 mosquitoes/day. Determine the population of mosquitoes in the area at any time.
Declare the definitions of your variables clearly with units.

What we're Given (w/ no Preds!)

$$\frac{dP}{dt} = kP, \quad \frac{P(t+1)}{P(t)} = 2 \quad t \in [0, \infty) \quad (t \text{ has units of weeks})$$

We know the solution is $P(t) = Ce^{kt}$, We use the above condition to find k:

$$\frac{P(t+1)}{P(t)} = \frac{e^{k(t+1)}}{e^{kt}} = e^{k} = 2 \iff k = \ln(2)$$

Solving the Question (w/ Preds)

Now we have that the population d es by $N = 20{,}000/\text{day}$

Our O.D.E becomes: (7 days in a week)

$$\frac{dP}{dt} = \ln(2)P - 7N$$

We know this has solution:

$$P(t) = \frac{7N}{\ln(2)} + C2^{t}$$

The initial condition that $P(0) = 200{,}000$ allows us to find C, it is

$$C = 200{,}000 - \frac{7 \cdot 20{,}000}{\ln(2)} \Rightarrow P(t) = \frac{7 \cdot 20{,}000}{\ln(2)} + \left(200{,}000 - \frac{7 \cdot 20{,}000}{\ln(2)}\right)2^{t}$$

5. Construct a first order linear differential equation such that all of its solutions are asymptotic to the line $y = 2 - t$ as $t \to \infty$. Then solve your equation and confirm that the solutions do indeed have the specified property.

Want that $\lim_{t \to \infty} [y(t) - (2-t)] = 0$

so something like $y(t) = t - 2 + \frac{c}{t}$

$y' = 1 - \frac{c}{t^2} \Rightarrow y' + \frac{1}{t} y = 2\left(1 - \frac{1}{t}\right)$

6. Let w be a function of x. Solve the following differential equation. You may leave the solution in implicit form.

$$xw'(x)\cos(3w(x)) = \frac{1}{1+x} + \cos(3w(x))w'(x)$$

$$\Leftrightarrow (x-1)w'\cos(3w) = \frac{1}{1+x}$$

$$\Leftrightarrow \int \cos(3w)\,dw = \int \frac{1}{x^2-1} = -\frac{1}{2}\int \frac{1}{1-x} + \frac{1}{x+1}$$

$$\Leftrightarrow \frac{\sin(3w)}{3} = \frac{1}{2}\left(\ln(1-x) - \ln(1+x)\right) + C$$

$$\Leftrightarrow w(x) = \frac{1}{3}\arcsin\left(\frac{3}{2}\ln\left(\frac{1-x}{1+x}\right) + C\right)$$